

Hong Kong Mathematics Olympiad (2011 / 2012)
Heat Event (Group)
香港数学竞赛 (2011 / 2012)
初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 已知 x 、 y 及 z 为三个连续正整数，且 $\frac{y}{x} + \frac{z}{x} + \frac{x}{y} + \frac{z}{y} + \frac{x}{z} + \frac{y}{z}$ 为整数，求 $x + y + z$ 的值。

Given that x , y and z are three consecutive positive integers, and $\frac{y}{x} + \frac{z}{x} + \frac{x}{y} + \frac{z}{y} + \frac{x}{z} + \frac{y}{z}$ is an integer.

Find the value of $x + y + z$.

2. 已知 x 是一个实数，且 $\sqrt{x-2012} + \sqrt{(5-x)^2} = x$ ，求 x 的值。

Given that x is a real number and $\sqrt{x-2012} + \sqrt{(5-x)^2} = x$. Find the value of x .

3. 求 $\sqrt{2^2 + 2^{1008} + 2^{2012}}$ 的值。(答案可以指数表示。)

Evaluate $\sqrt{2^2 + 2^{1008} + 2^{2012}}$. (Answer can be expressed in index form.)

4. 求 $\frac{1}{\sqrt{2012} + \sqrt{2011}} + \frac{1}{\sqrt{2011} + \sqrt{2010}} + \cdots + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{1}}$ 的值。(答案可以根式表示。)

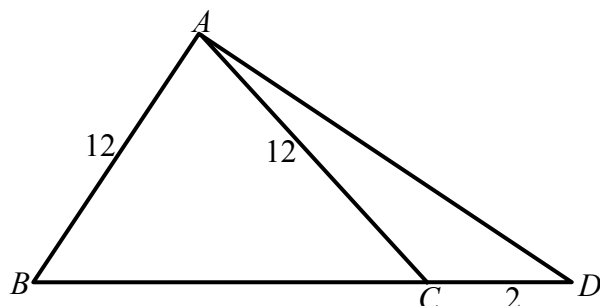
Evaluate $\frac{1}{\sqrt{2012} + \sqrt{2011}} + \frac{1}{\sqrt{2011} + \sqrt{2010}} + \cdots + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{1}}$. (Answer can be expressed in surd form.)

5. 求 $x^2 + y^2 - 10x - 6y + 2046$ 的最小值。

Find the minimum value of $x^2 + y^2 - 10x - 6y + 2046$.

6. 如图三, $\triangle ABC$ 为一等腰三角形。设 $AB = AC = 12$ 。若 D 是 BC 延伸在线的一点, 使 $\angle DAB = 90^\circ$ 及 $CD = 2$, 求 BC 的长。

In Figure 3, $\triangle ABC$ is an isosceles triangle. Suppose $AB = AC = 12$. If D is a point on the BC produced such that $\angle DAB = 90^\circ$ and $CD = 2$, find the length of BC .



图三
Figure 3

7. 已知 $a^x = b^y = c^z = 30^w$ 及 $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{w}$, 当中 a, b, c 为正整数 ($a \leq b \leq c$) 及 x, y, z, w 为实数。求 $a + b + c$ 的值。

Given that $a^x = b^y = c^z = 30^w$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{w}$, where a, b, c are positive integers ($a \leq b \leq c$) and x, y, z, w are real numbers, find the value of $a + b + c$.

8. 已知方程 $x^2 + px + q = 0$ 的两个根为整数, 且 $q > 0$ 。若 $p + q = 60$, 求 q 的值。

Given that the roots of the equation $x^2 + px + q = 0$ are integers and $q > 0$. If $p + q = 60$, find the value of q .

9. 求 $\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \cdots + \sin^2 359^\circ + \sin^2 360^\circ$ 的值。

Evaluate $\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \cdots + \sin^2 359^\circ + \sin^2 360^\circ$.

10. 在一集会中，原先安排每位宾客与其他宾客各握手一次，但小明只和他认识的人握手。如果集会中实际握手的总次数为 60 次，那么小明在该集会中认识多少人？(注：当两人相互握手，握手的总次数会是一次（而不是两次）。)

In a gathering, originally each guest will shake hands with every other guest, but Steven only shakes hands with people whom he knows. If the total number of handshakes in the gathering is 60 , how many people in the gathering does Steven know? (Note: When two persons shake hands with each other, the total number of handshakes will be one (not two).)

END